Metrol. Meas. Syst., Vol. XVII (2010), No. 2, pp. 00–00

METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229

www.metrology.pg.gda.pl

INVESTIGATING ADVANTAGES AND DISADVANTAGES OF THE ANALYSIS OF A GEOMETRICAL SURFACE STRUCTURE WITH THE USE OF FOURIER AND WAVELET TRANSFORM

Stanisław Adamczak, Włodzimierz Makieła, Krzysztof Stępień

Kielce University of Technology, Faculty of Mechatronics and Machinery Design, Al. 1000-lecia P.P. 7, 25–314 Kielce, Poland (adamczak@tu.kielce.pl, wmakiela@tu.kielce.pl, ✉ kstepien@tu.kielce.pl, +48 41 342 4477*)*

Abstract

Nowadays a geometrical surface structure is usually evaluated with the use of Fourier transform. This type of transform allows for accurate analysis of harmonic components of surface profiles. Due to its fundamentals, Fourier transform is particularly efficient when evaluating periodic signals. Wavelets are the small waves that are oscillatory and limited in the range. Wavelets are special type of sets of basis functions that are useful in the description of function spaces. They are particularly useful for the description of non-continuous and irregular functions that appear most often as responses of real physical systems. Bases of wavelet functions are usually well located in the frequency and in the time domain. In the case of periodic signals, the Fourier transform is still extremely useful. It allows to obtain accurate information on the analyzed surface. Wavelet analysis does not provide as accurate information about the measured surface as the Fourier transform, but it is a useful tool for detection of irregularities of the profile. Therefore, wavelet analysis is the better way to detect scratches or cracks that sometimes occur on the surface. The paper presents the fundamentals of both types of transform. It presents also the comparison of an evaluation of the roundness profile by Fourier and wavelet transforms.

Keywords: wavelet, Fourier transform, geometrical surface structure.

1. Introduction

When designed, a machine element is assumed to be ideal in shape and texture. The surface must be ideally smooth, too. However, numerous factors cause that the geometrical surface structure of a workpiece is characterized by certain irregularities. These irregularities lead to differences between the real and the nominal workpiece. Fig. 1 shows the irregularity types.

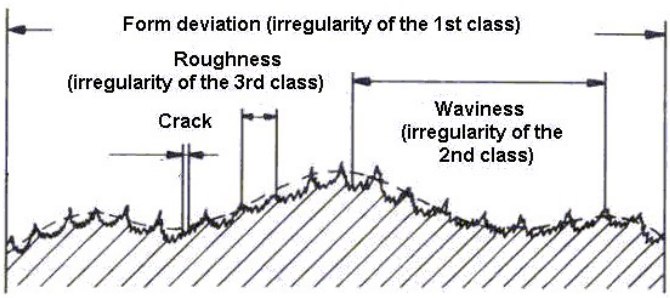

Fig. 1. Types of surface irregularities [1].

– surface roughness, which occurs when the distance between irregularities is from about 5 to about 50 times larger than their depth. Surface roughness is usually related to tool paths, for example, turning, grinding or finishing cuts [2, 3];

Article history: received on Apr. 26, 2010; accepted on Jun. 8, 2010; available online on Jun. 16, 2010.

- waviness, which is observed when the distance between irregularities is from about 50 to about 1000 times larger than their depth, in some sources the lower limit is 40. Waviness is usually the result of a disturbed manufacturing process [4], an example of which can be vibrations between the workpiece and the grinding wheel;
- form deviations are assumed to occur when the distance between irregularities is equal or higher than 1000:1. Shape deviations are caused, for example, by guideway errors, errors of rotary elements, or by thermal expansion of elements [5].

It should be noted that cracks may also be considered as a type of irregularity, for which the ratio between the distance and the depth is less than or equal to 5.

2. The application of Fourier transform to the analysis of geometrical surface structure

Within the area of surface metrology, measured profiles are often periodic ones [6]. An example can be roundness profiles described by the polar coordinates R and φ, where R is the distance of the point from the rotation axis and φ is the angular coordinate of this point. Each roundness profile can be approximated by the so-called Fourier series, which in trigonometric form is the following:

$$R(\phi) = R_0 + \sum_{n=1}^{\infty}(a_n \cos n\phi + b_n \sin n\phi). \tag{1}$$

The coefficients of the series can be calculated as follows:

$$R_0 = \frac{\int_0^{2\pi} R(\varphi)d\varphi}{2\pi},\ a_n = \frac{1}{\pi}\int_0^{2\pi} R(\varphi)\cos n\varphi d\varphi,\ b_n = \frac{1}{\pi}\int_0^{2\pi} R(\phi)\sin\phi d\phi. \tag{2}$$

For roundness profiles the meaning of subsequent Fourier coefficients can be explained as follows: the coefficient no. 0 relates to the mean radius of the measured element, the coefficients no. 1 relate to the eccentricity of the element, the coefficients no. 2 describe the error of ovality, the coefficients no. 3 relate to the error of triangularity of the profile, *etc.*
Continuous Fourier transform $F(\omega)$ and the inverse transform $f(t)$ can be calculated from the formulas (3–4):

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt, \tag{3}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}dt. \tag{4}$$

In practice, in digital signal processing, a so-called Fast Fourier Transform algorithm is used. It is an algorithm for efficient computing of a discrete Fourier transform (DFT). It can be used also for computing the inverse transform. The DFT can be computed by the following formula for x_0 to x_{N-1} complex numbers:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-ikn\theta}, \tag{5}$$

where $k = 0,\ldots,N-1$, and the sampling period $\theta = \frac{2\pi}{N}$.

The following diagrams show the application of the Fourier transform to the evaluation of two roundness profiles and comparison of the profiles with use of their harmonic components.

In Fig. 2 roundness profiles are shown in Cartesian coordinates.

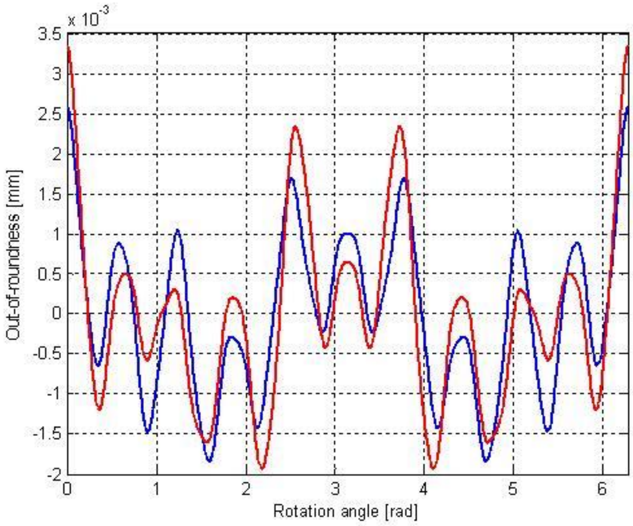

Fig. 2. Plots of compared roundness profiles in Cartesian coordinates.

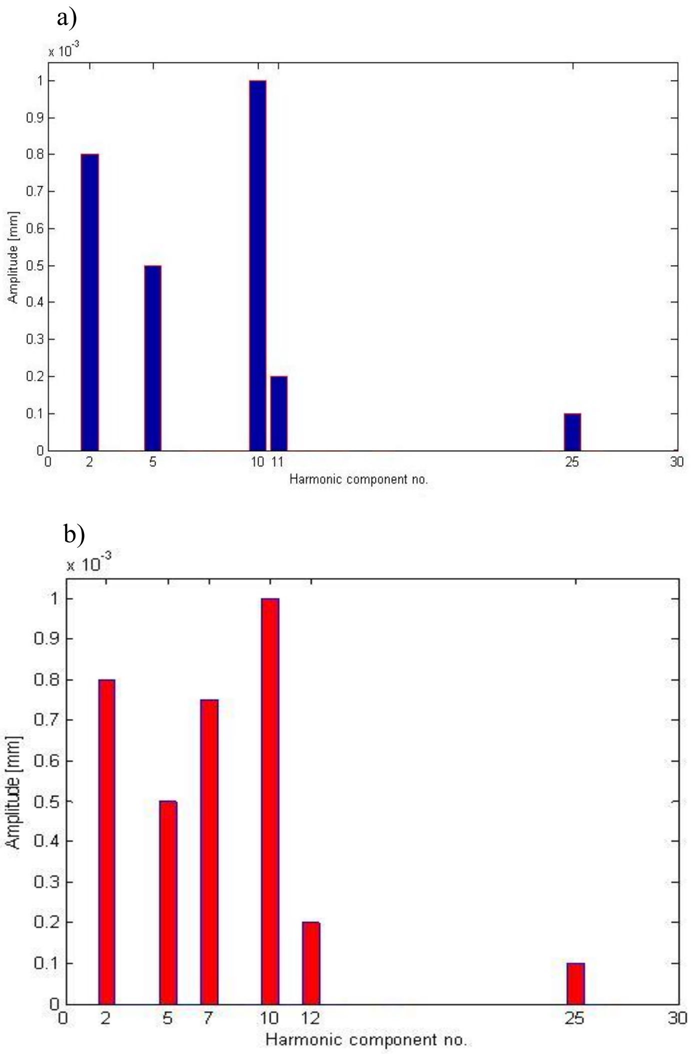

Fig. 3. Comparison of harmonic components of the profiles shown in Fig. 2.

In order to find the difference of harmonic components of both profiles, the Fourier transform has been applied. Practically, the so-called Fast Fourier Transform (FFT) has been conducted. Fig. 3 presents harmonic components of the compared profiles. It is easy to notice that the profiles differ in occurrence of the harmonic component no. 7, 11 and no. 12.

3. Wavelet transform and its application for the analysis of a geometrical surface structure

Wavelets $\psi(t)$ are small waves that are oscillatory and limited in range [7, 8]. Independent variable t is defined as "time" or sometimes "a spatial variable". Wavelets are special type of sets of basis functions that are useful in the description of function spaces. They are particularly useful for description of non-continuous and irregular functions that appear most often as responses of real physical systems. Bases of wavelet functions are usually well located in the frequency and the time domain. They are formed by scaling (with the parameter a) and time shifting - the translations (with the parameter b) of the mother wavelet $\psi(at + b)$ [9]. It leads to a so-called scaled, hierarchic representation of the investigated function [10]. The mother wavelet was for the first time applied by J. Morlet. It was the first practically used system of orthogonal decomposition of signals to the time-frequency domain, which allowed for discovering the new area of research, as signal processing in the time-frequency domain [11].

An important problem in modern engineering is the analysis of a measured signal. The aim that should be achieved by the signal analysis, is obtaining accurate information on the most important features of the element or detection of damages [12]. Wavelet analysis allows for representation of the signal in the base of wavelet functions, that are compact and local functions, not global ones as it was in the case of Fourier analysis. Each wavelet function is located at a different spot along the time axis and its range depends on the range of applied scales [13].

For a continuous signal, the wavelet transform can be computed from the formula:

$$\tilde{s}_{\psi}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(t)\psi\left(\frac{t-b}{a}\right) dt\,, \tag{6}$$

where: a – scale, b – shift, $s(t)$ – signal in time, ψ – wavelet function.

One of the wavelet transform types is recurrent computing of sums and differences of elements for all scale levels. It is a simple type of filtration that can replace more complex filtration which leads to wavelets generating better transforms [12]. Replacing subsequent component couples x_{i-1} and x_i of the vector x by their sums y_i that are components of the new signal y is equivalent to averaging or smoothing of this signal. In order to avoid changing the level of the signal this sum is divided by two. Equation (7) is the equation of the simplest low-pass filter whose current output is the average from the last two values of the input signal.

$$y_i = \frac{1}{2}x_i + \frac{1}{2}x_{i-1}. \tag{7}$$

The twin filter is the high-pass filter, which generates the current difference of last two values of the input signal. The output signal of the high-pass filter represents the fluctuations of the input signal determined by the high frequency components. The equation of the high-pass filter is the following:

$$y_i = \frac{1}{2}x_i - \frac{1}{2}x_{i-1}. \tag{8}$$

The algorithm of the input signal decomposition with the use of both types of filters is shown in Fig. 4.

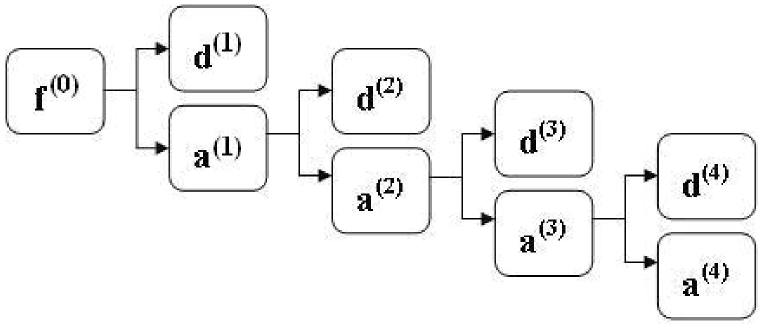

Fig. 4. The tree of wavelet decomposition: $f^{(0)}$ – measured signal, $a^{(i)}$ – an approximation in the *i*-th – step, $d^{(i)}$ – addition (detail) in the ***i-th*** – step.

In the following Fig. 5 the decomposition of the signal with use of the Haar wavelet is shown. The Haar wavelet is a relatively simple wavelet that is mainly used for didactic purposes. It is the only wavelet that has the property of orthogonality and the axis of asymmetry. It means that the corresponding filter does not provide a nonlinear shift between the input and output signals. It allows, for example, a cascade connection of filters without any compensation of the phase of the signal.

A Haar wavelet is defined by the following equations:

$$\Psi(t) = \begin{cases} 1 & \text{for } 0 \le t < 0{,}5 \\ -1 & \text{for } 0{,}5 \le t < 1 \\ 0 & \text{for other } t. \end{cases} \tag{9}$$

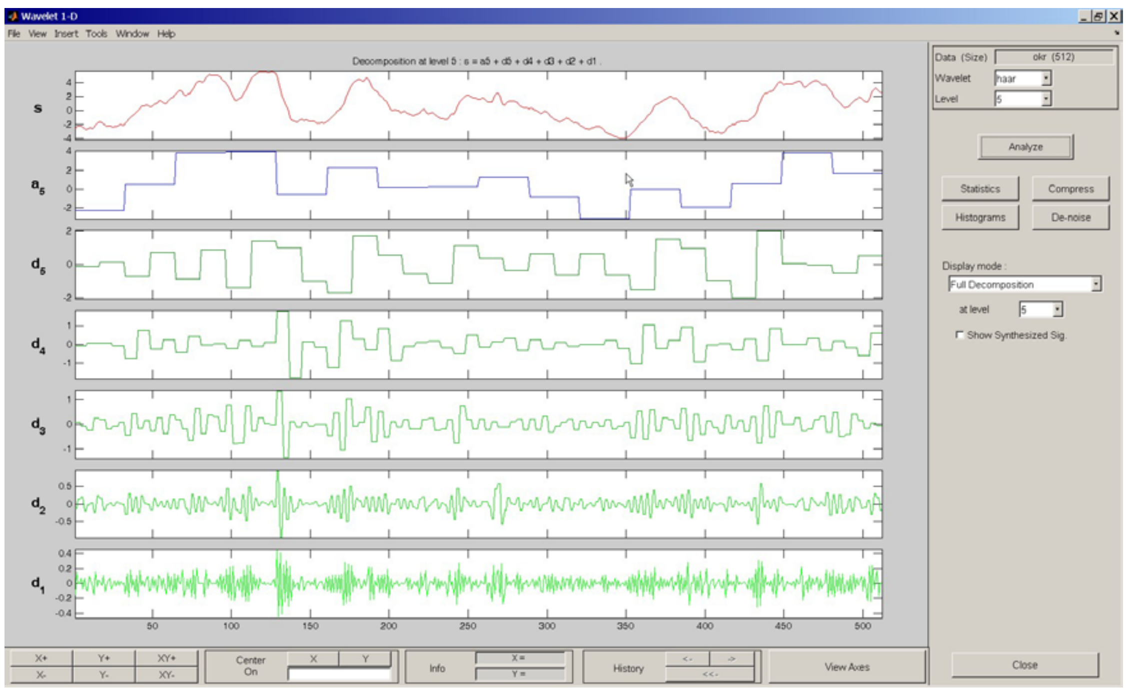

Fig. 5. Decomposition of measured signal s; a_5 – an approximation on the 5th level, d_i – additions (details) on the *i*-th level of decomposition.

The most popular types of wavelets that are used in signal analysis are the following: Daubechies, Morlet, Meyer, Coilfets, Symlets, “Mexican hat” and biorthogonal wavelets.

3.1. Denoising and signal reconstruction with the use of wavelets

Wavelet transformation is a lossless process meaning that we can decompose the analyzed signal, and then using the wavelet coefficients in the reconstruction process acquire a signal identical to the original one.

Below an example of the denoising process is presented. The process was conducted with the use of the Wavelet Toolbox being a component of the Matlab package.

As a result of the multiple decomposition process (by using the wavedec function) one can construct a vector which contains the following two elements: the last approximation coefficients and all of the signal details. Reconstruction of approximation of signal A_0 is carried out using the waverec function. The multiple decomposition process allows us to use the thresholding method for signal denoising.

Signal denoising can be performed using MATLAB Wavelet Toolbox. In order to denoise the signal the following steps need to be taken:

– import the analyzed signal and choose the wavelet as well as the decomposition level K;
– decompose the signal using the specific wavelet and level chosen in the previous step;
– set the threshold value throughout the decomposition levels from 1 to K and remove the appropriate details;
– reconstruct the signal using original approximation coefficients from level K and the modified details from levels 1 to K.

Furthermore, it should be noted that there are two possible threshold methods to choose from:

– hard thresholding which excludes all the details that are smaller than the threshold value leaving other details unchanged;
– soft thresholding which excludes all the details that are smaller than the threshold value and decreases the remaining details by the threshold value.

We can also perform a selective reconstruction. This procedure comes down to excluding details from a chosen level and can also be accepted as a rough denoising method. It resembles the filtration process carried out using Fourier transform where a chosen oscillation can be omitted.

In order to evaluate the efficiency with which the two transforms denoise signals, we conduct the following test. We distorted the measured waviness profile by adding white noise, and then used both transforms to denoise it.

The experiment results for wavelet transform can be viewed in Fig. 6 (6a: the measured signal, 6b: distorted signal, 6c: signal approximation on level five). Comparative calculations of signals 6a and 6c show very high denoising efficiency of the wavelet transform. This can also be easily noticed on the graphs presented in Fig. 6.

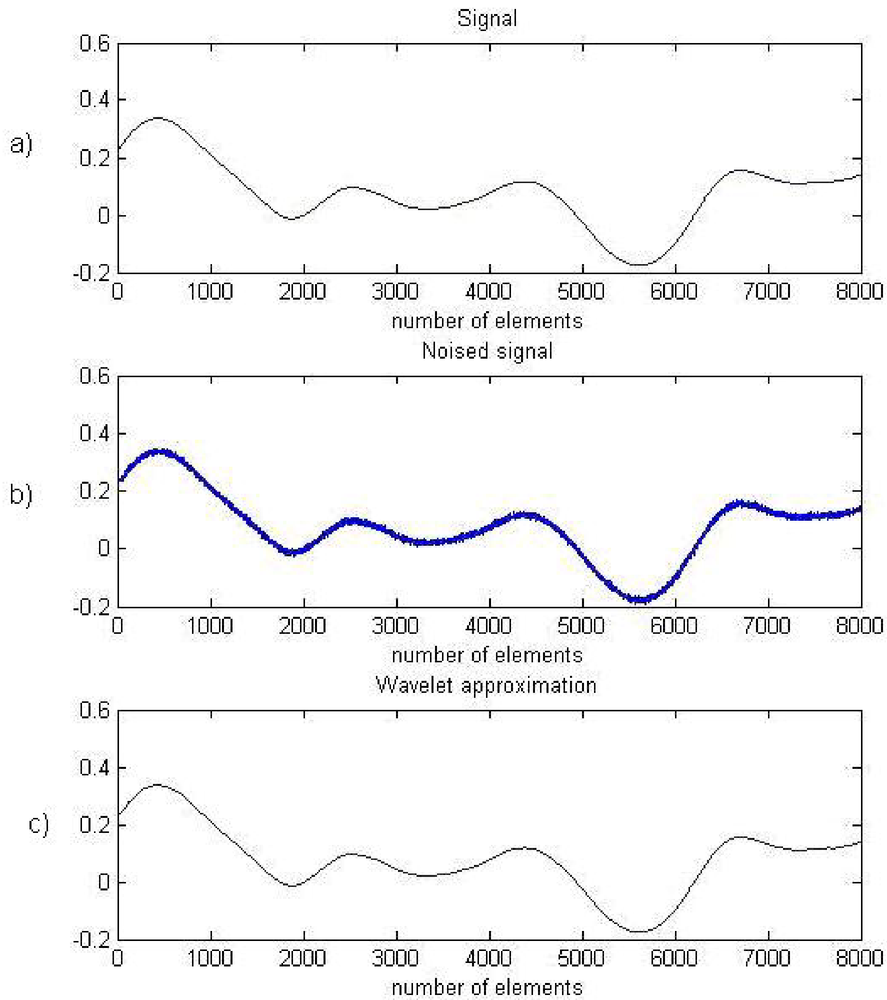

Fig. 6. Denoising simulation process; white noise removal using db5 wavelet transform; a) the measured signal, b) distorted signal, c) level five signal approximation.

4. Comparison of the application of Fourier and wavelet transforms to the evaluation of roundness profiles

In order to compare the results of the evaluation of roundness profiles by Fourier and wavelet transforms, the following roundness profile will be investigated:

$$R(\varphi) = 5 + \cos(\varphi) + \cos(2\varphi) + 0.5\cos(5\varphi) + \cos(10\varphi) + \cos(11\varphi) + \cos(25\varphi) \qquad (10)$$

From the relationship (10) one can conclude that the profile will be the combination of following harmonic components: the zero component with the amplitude equal to 5, the first component with the amplitude equal to 1, the components no. 2, 10, 11 and 25 equal to 1 and the fifth component with the amplitude equal to 0.5.

The plot of the profile given by formula (10) is shown in Fig. 7.

The profile shown in Fig. 7 can be easily evaluated by the Fourier transform because it is a combination of cosine curves. After conduction of the FFT algorithm the set of values of harmonic components of the profile that are shown in Fig. 8 was obtained. The harmonic components from the range 1–35 were analyzed.

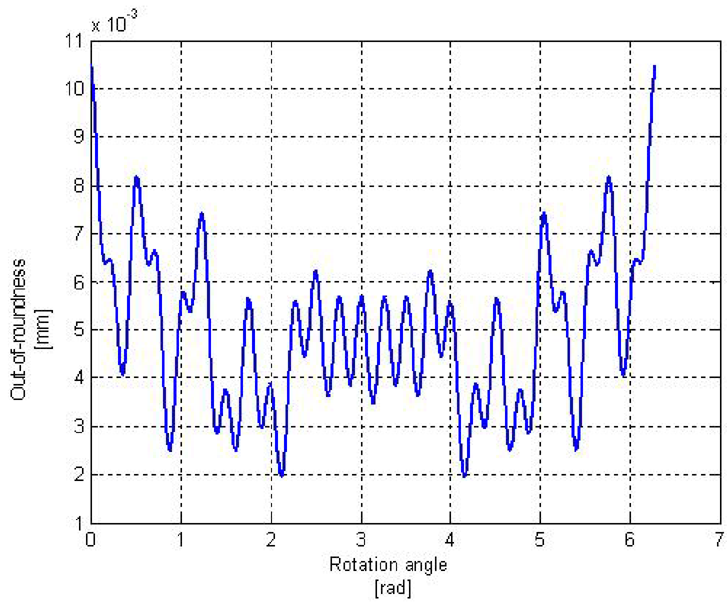

Fig. 7. The roundness profile given by formula (10).

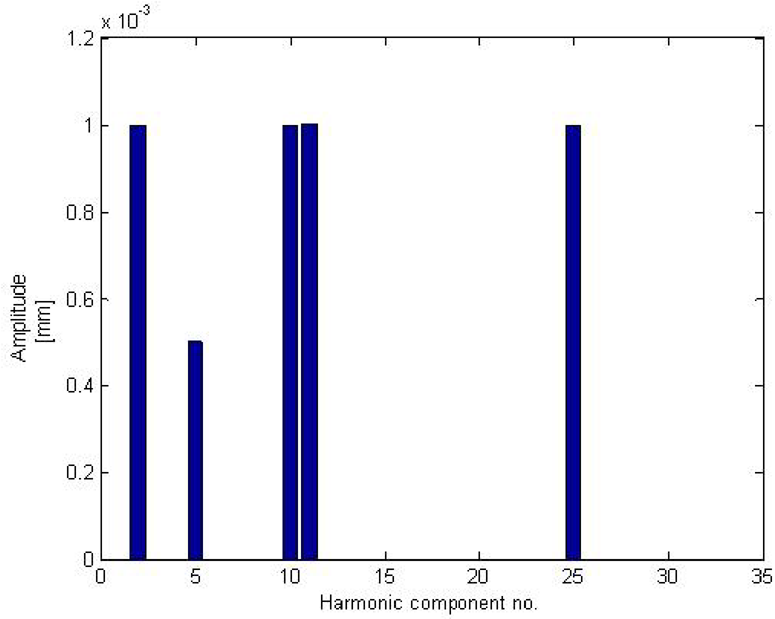

Fig. 8. Amplitudes of harmonic components of the profile shown in Fig. 7.

From Fig. 8 it is clear that Fourier analysis allowed to obtain accurate information on the investigated profile. The application of the FFT algorithm provided the values of the harmonic components that constitute the analyzed signal.

In the next stage of investigation the signal given by the formula (10) will be disturbed by introducing three zero values approximately into the centre of the profile. Such disturbance can simulate, for example, a scratch on the real surface. The generated profile is shown in Fig. 9.

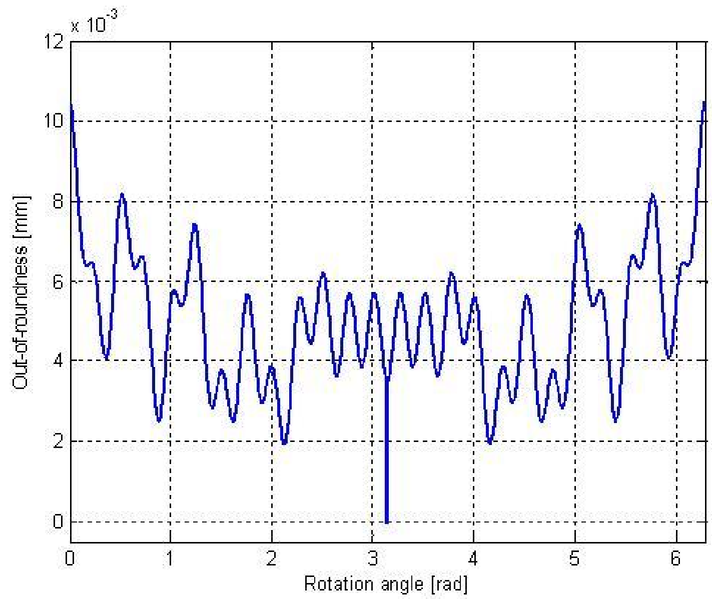

Fig. 9. The profile containing the disturbance.

After conducting the FFT algorithm one can obtain the following diagram of the amplitudes of harmonic components of the profile.

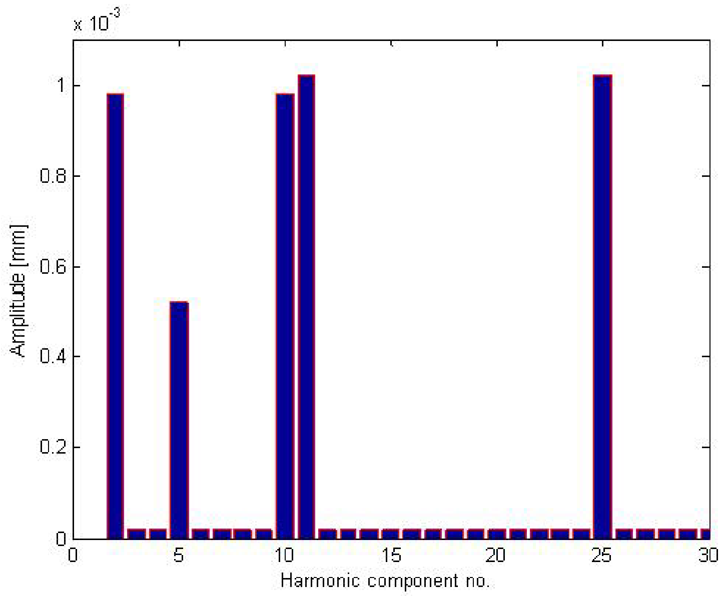

Fig. 10. Analysis of harmonic components of the signal shown in Fig. 9.

It is easy to notice that in this case the application of the Fourier transform does not allow to obtain accurate information on the characteristics of the analyzed signal.

In order to investigate if the wavelet analysis can provide more accurate results, the wavelet transform with use of a discrete Meyer wavelet was conducted. In the transform five levels of the decomposition were applied.

It is obvious that conducting the wavelet analysis does not provide such accurate information about the analyzed profile as it is in case of the Fourier transform. However, in the analyzed case the wavelet analysis allowed the detection of the disturbance in the investigated profile. This disturbance is easy to notice particularly when analyzing the diagrams of the coefficients. It is also possible to define clearly where such disturbance occurs. Such information could not be obtained when the Fourier transform was applied (see Fig. 10).

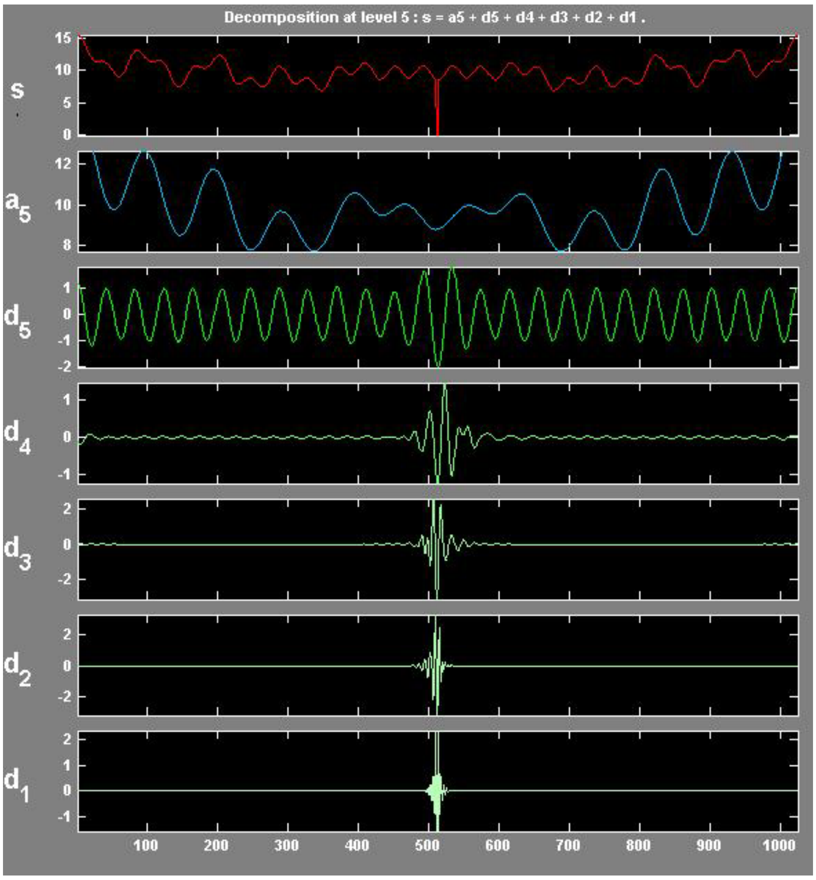

Fig. 11. Wavelet decomposition of the profile shown in Fig. 9 (type of mother wavelet – discrete Meyer wavelet, number of decomposition levels – 5).

5. Summary

The Fourier transform is extremely useful when analyzing periodic signals. Therefore it is a very useful tool for evaluation of roundness or cylindricity profiles. It usually allows to obtain accurate information on the analyzed surface. Wavelet transform does not provide such accurate information. However, because it is well localized in the time and frequency domains it can detect irregularities of the profile such as cracks or scratches of the surfaces. Wavelet transform is also a very convenient tool for denoising the measuring signal.

The example presented in this paper shows that if there is a scratch or other significant disturbance of the profile, the results of application of the Fourier transform can be sometimes doubtful. Such disturbances can be detected very easily when the wavelet transform is applied. Therefore one can conclude that both types of transforms can complement each other when evaluating geometrical surface structures.

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